ME 321: Fluid Mechanics-I

Prof. Dr. A.B.M. Toufique Hasan
Department of Mechanical Engineering
Bangladesh University of Engineering and Technology (BUET)

Lecture - 04 (03/05/2025)
Fluid Dynamics: Streamlines & RTT

toufiquehasan.buet.ac.bd
toufiquehasan@me.buet.ac.bd

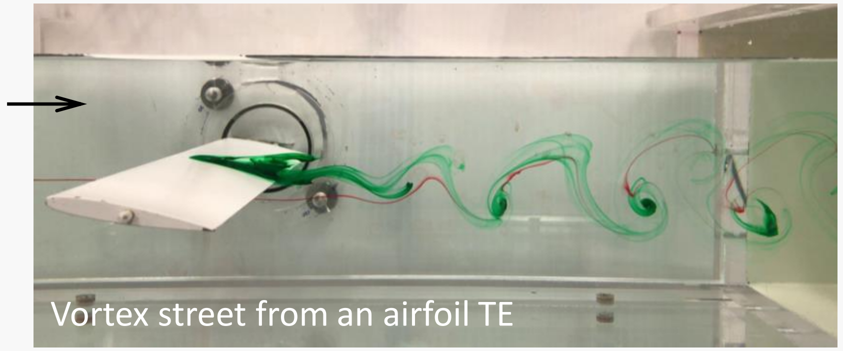
Visualization of Fluid flows

Flow visualization is the technical art of making flow patterns visible.
It is a fundamental technique to describe and understand the nature of fluid flow dynamics in and around a flow system (both internal and external flows) and thus grossly predict its performance.

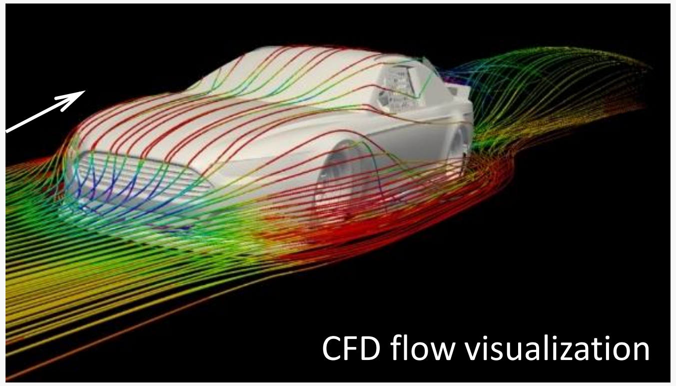
Experimental flow visualization technique includes dye visualization (streakline, pathline), smoke visualization, particle image velocimetry (PIV), particle tracking velocimetry (PTV), surface flow visualization (Pressure sensitive paint, etc.), Schlieren imaging, shadowgraph, laser diagnosis, Mie scattering, and so on.

In **analytical fluid dynamics**, the visualization of fluid flow is usually conducted using streamline, contours of different flow variables, etc.

Karman Vortex street



Vortex street from an airfoil TE



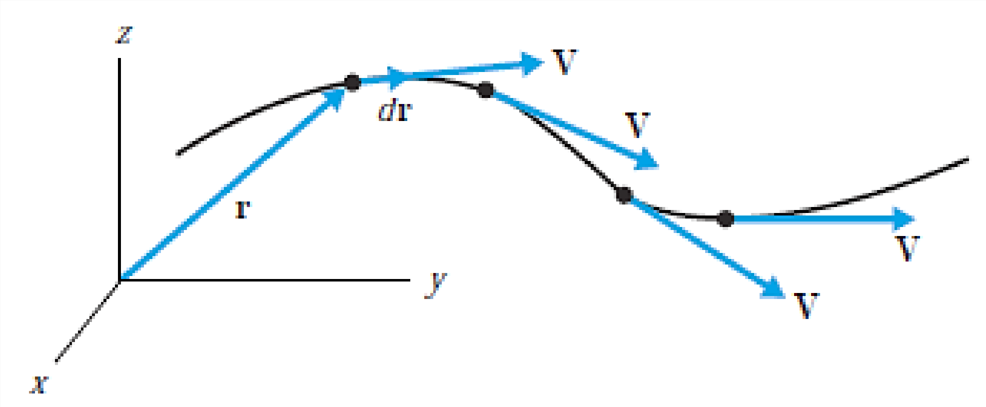
CFD flow visualization

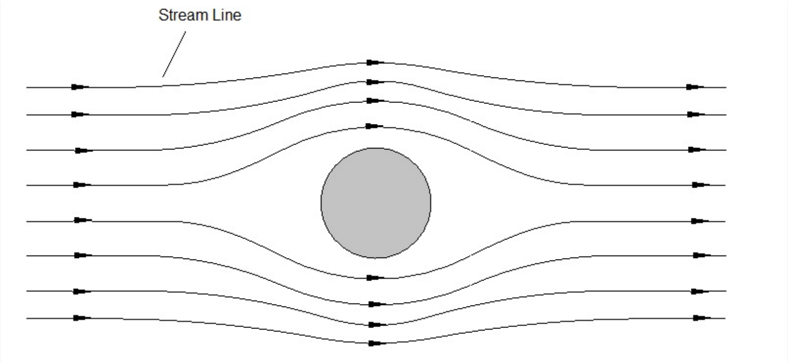
Streamline

A **streamline** is an imaginary line (curve) drawn through the flowing fluid in such a way that the tangent to it at any point gives the direction of the velocity at the point. Streamlines can not cross each other. **Streamline is often used in analytical work in fluid dynamics.**

Streamlines can be obtained analytically by integrating the equations defining lines tangent to the velocity field.

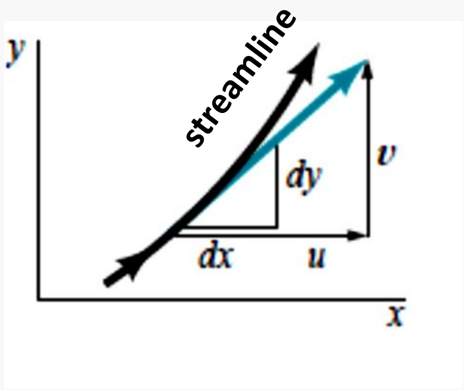
For 2-D flows, the slope of the streamline must be equal to the tangent of the angle that velocity vector makes with x-axis.

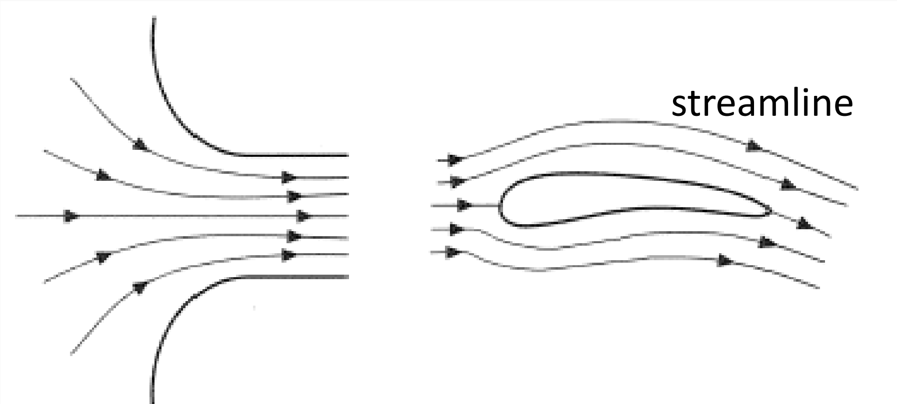




Flow over circular cylinder

$$\boxed{\frac{dy}{dx} = \frac{v}{u}}$$





Flow into pipe intake and flow over airfoil

If the velocity field is known as a function of x and y (and t if the flow is unsteady), this equation can be integrated to give the equation of the streamlines.

Problem

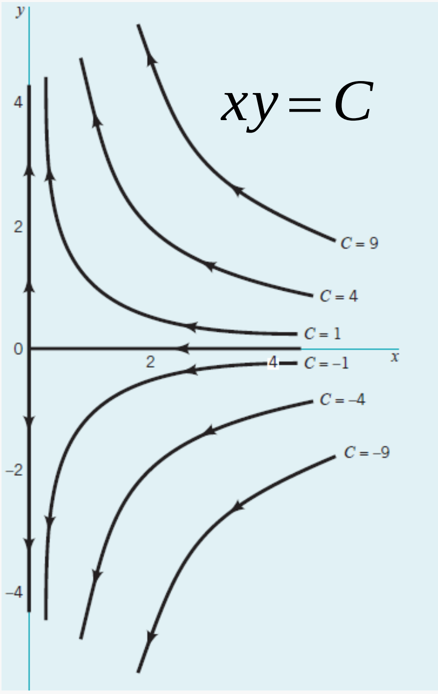
A velocity field is given by

$$\mathbf{V} = \left(\frac{V_0}{l}\right)\left(-x\hat{i} + y\hat{j}\right)$$

where V_0 and l are constants. Determine

(i) Streamlines for this flow

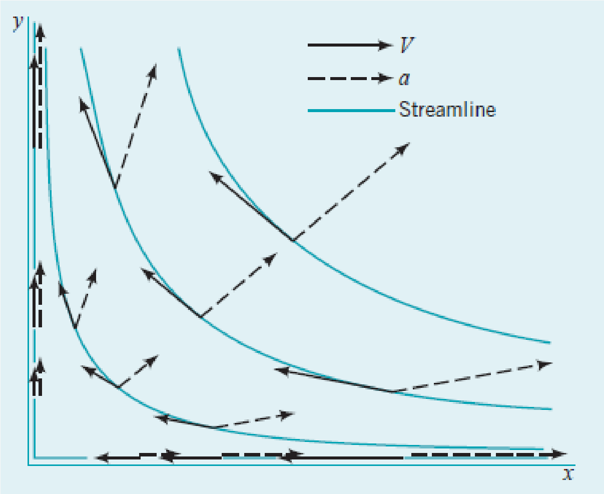
(ii) Acceleration field for this flow.



Answer:

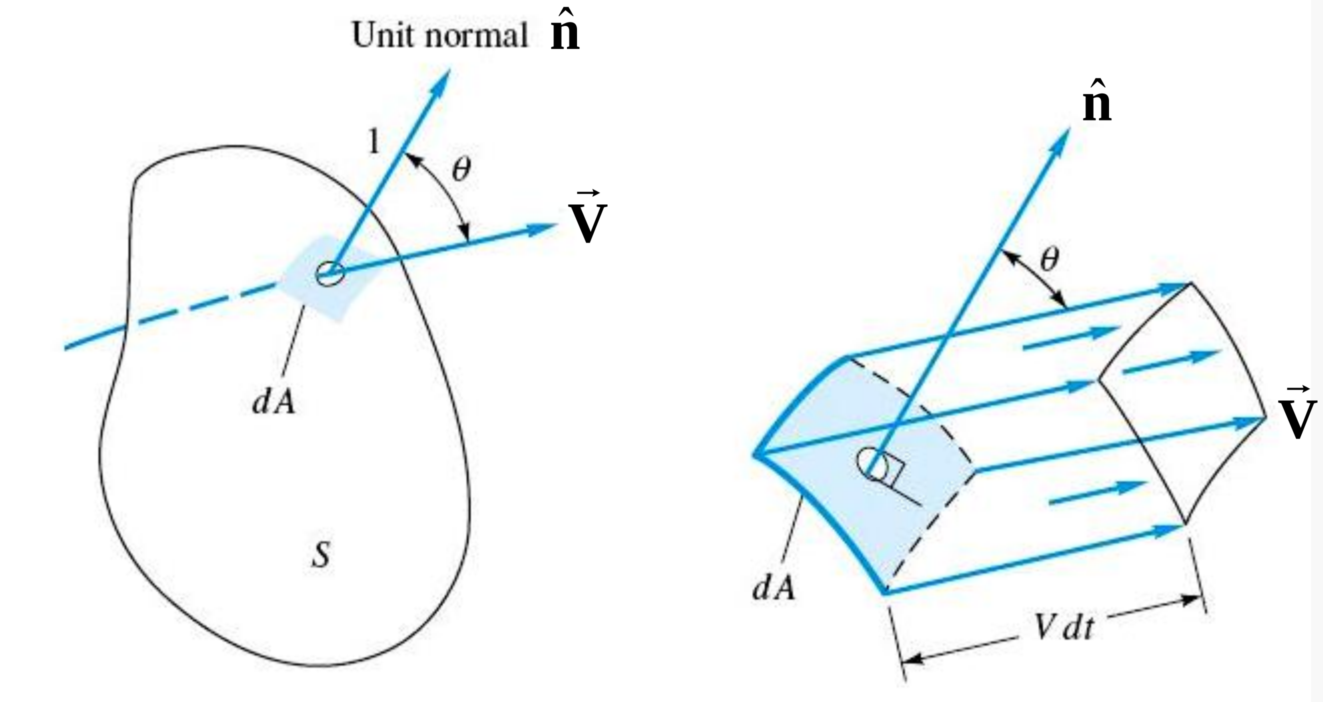
$(i)\, xy = C$

$(ii)\; a_x = \dfrac{V_0^2 x}{l^2},\; a_y = \dfrac{V_0^2 y}{l^2}$



Fluid Flow Rate

Suppose S is an arbitrary surface through which fluid is flowing without resistance. We need to determine rate of amount of fluid flows.



Typically $\vec{\mathbf{V}}$ may pass through dA (elemental area of S) at angle θ off the normal. **Let $\hat{\mathbf{n}}$ be the unit vector normal outward to dA**. Then the amount of fluid swept through dA in time dt is the volume of the slanted parallelepiped:

$$d\mathcal{V} = (V\,dt)(dA\cos\theta) = (\vec{\mathbf{V}}\cdot\hat{\mathbf{n}})\,dA\,dt$$

$$\therefore \frac{d\mathcal{V}}{dt} = (\vec{\mathbf{V}}\cdot\hat{\mathbf{n}})\,dA \equiv dQ$$

Total **volume flow rate**, $Q = \int_S dQ = \int_S (\vec{\mathbf{V}}\cdot\hat{\mathbf{n}})\,dA = \int_S V_n\,dA$

V_n is the component of $\vec{\mathbf{V}}$ normal to dA

cusec (ft^3/s), cumec (m^3/s), m^3/hr, lit/min, etc.

Total **mass flow rate**, $\dot{m} = \int_S \rho(\vec{\mathbf{V}}\cdot\hat{\mathbf{n}})\,dA = \int_S \rho V_n\,dA$

If density and velocity are constant over the surface S, a simple expression results:

$$\boxed{\dot{m} = \rho Q = \rho A V}$$

***A* and *V* must be perpendicular**

Analysis of Fluid Dynamics

In analytical fluid dynamics, there are two approaches for the solution of flow problems:

(1) Detail description of flow pattern at every point (x, y, z) in the flow field
– known as **differential analysis (differential relations, differential equations)**

(2) Working with a finite region **(control volume (CV))**, making a balance of flow-in versus flow-out, and determining the **gross flow effects** such as the force or torque on a body or the total energy exchange.
– known as **integral analysis (integral relations, integral equations)**

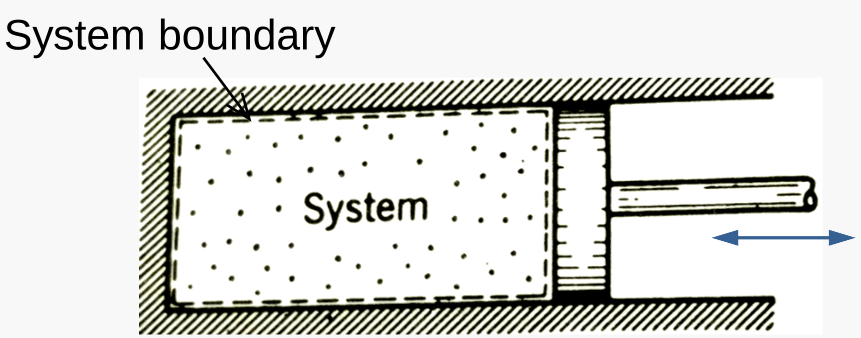
Integral relations will be covered first in this course.

System & Control Volume

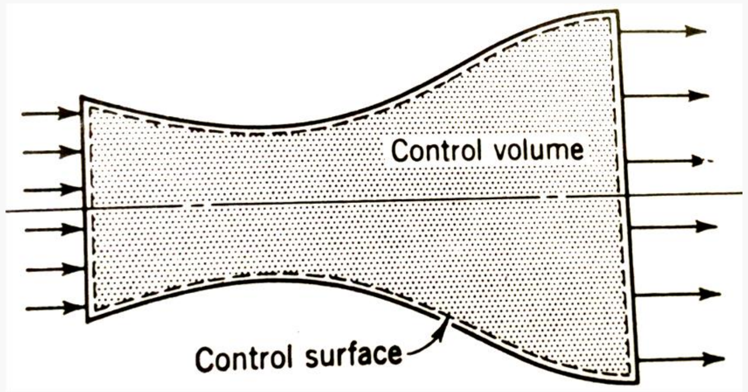
To analyze thermo-fluid dynamic problems, also there exist two approaches:

1. **System approach**: A system is defined as an arbitrary quantity of **fixed mass** (same atoms of fluid particles). A system may change shape, position, and thermal conditions but must entail the same amount of mass. Thus the mass of the system is conserved and does not change(except nuclear reactions). Everything external to this system is denoted by the term surroundings, and the system is separated from its surroundings by its boundaries.

2. **Control volume approach**: This approach concerns about the **fixed and definite volume in space (a geometric entity, independent of mass)**, known as ***control volume (CV)***. The boundary of this volume is known as ***control surface (CS)***. The amount of the matter in the control volume may change in time, but the shape of the control volume will remain fixed.



System approach

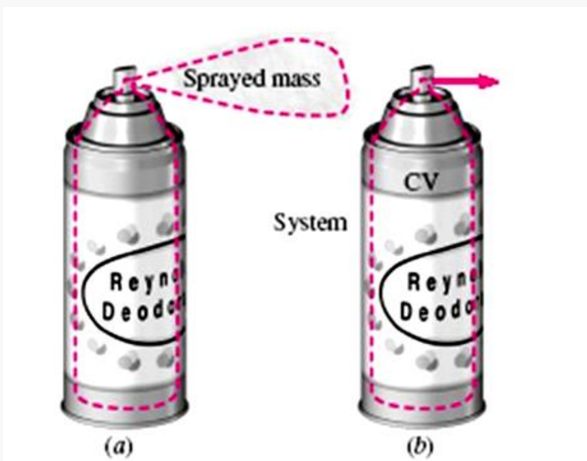


Control volume approach

System & Control Volume

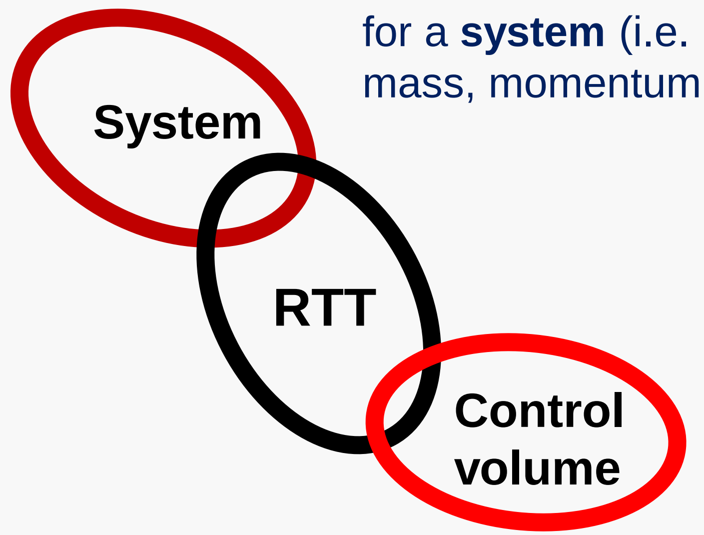
Sometimes we are interested in what happen to a particular part (amount) of the fluid when it moves. Other times, we may be interested in what effect the fluid has on a particular object or volume in space as fluid interact with it.

Thus, we need to describe the laws governing fluid motion using both system concepts (consider a fixed amount of mass of the fluid) and control volume concepts (consider a finite volume). To do this we need an analytical tool to shift from one representation to the other presentation. The **Reynolds Transport Theorem (RTT)** provides this tool.



System deformable

CV fixed, nondeformable



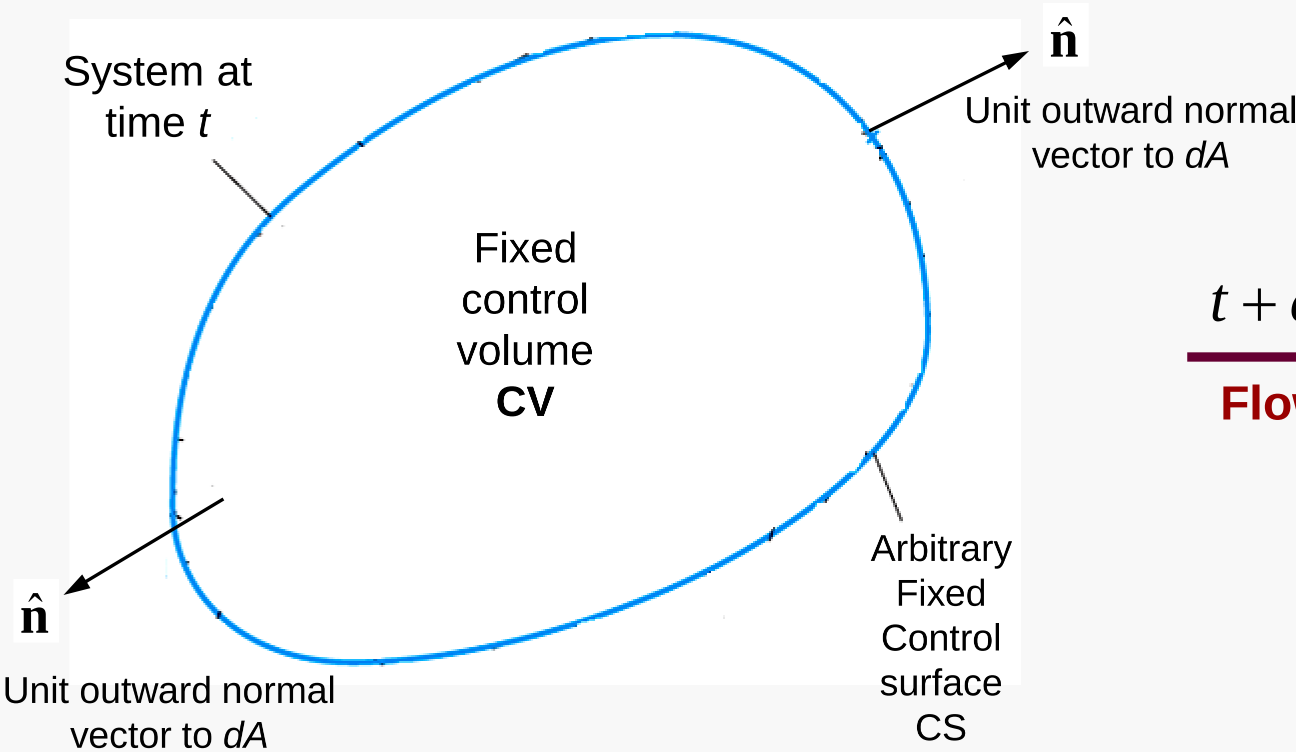
All the laws of Mechanics are written for a **system** (i.e. conservation of mass, momentum and energy)

Reformulation of the governing equations to **CV** approach through RTT

Reynolds Transport Theorem (RTT)

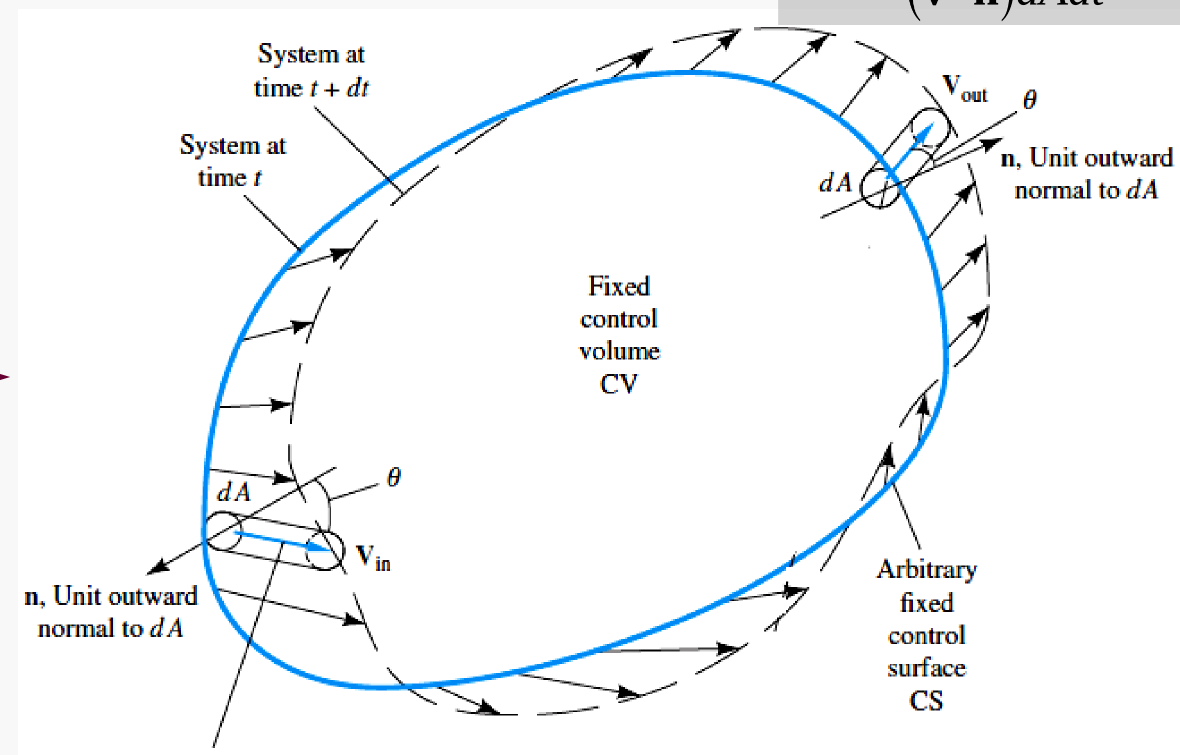
Flow out +ve

$$d\forall_{out} = V_{out}dA_{out}\cos\theta_{out}dt$$
$$= \left(\vec{\mathbf{V}}\cdot\hat{\mathbf{n}}\right)dAdt$$



$t + dt$

Flow



$$d\forall_{in} = V_{in}dA_{in}\cos\theta_{in}dt$$
$$= -\left(\vec{\mathbf{V}}\cdot\hat{\mathbf{n}}\right)dAdt$$

Flow in -ve

At time *t*,

both the system and control volume (CV) are the same identity.

Situation at time *t+dt*

Reynolds Transport Theorem (RTT)

Let *B* be any property of the fluid (mass, momentum, energy, enthalpy, etc.) and β is the intensive value of *B* (*dB*/*dm*) i.e. the amount of *B* per unit mass in any small element of the fluid.

The total amount of B in the fixed control volume (CV) is thus

$$B_{CV} = \int_{CV} \beta\, dm = \int_{CV} \beta\, \rho d\forall$$

where

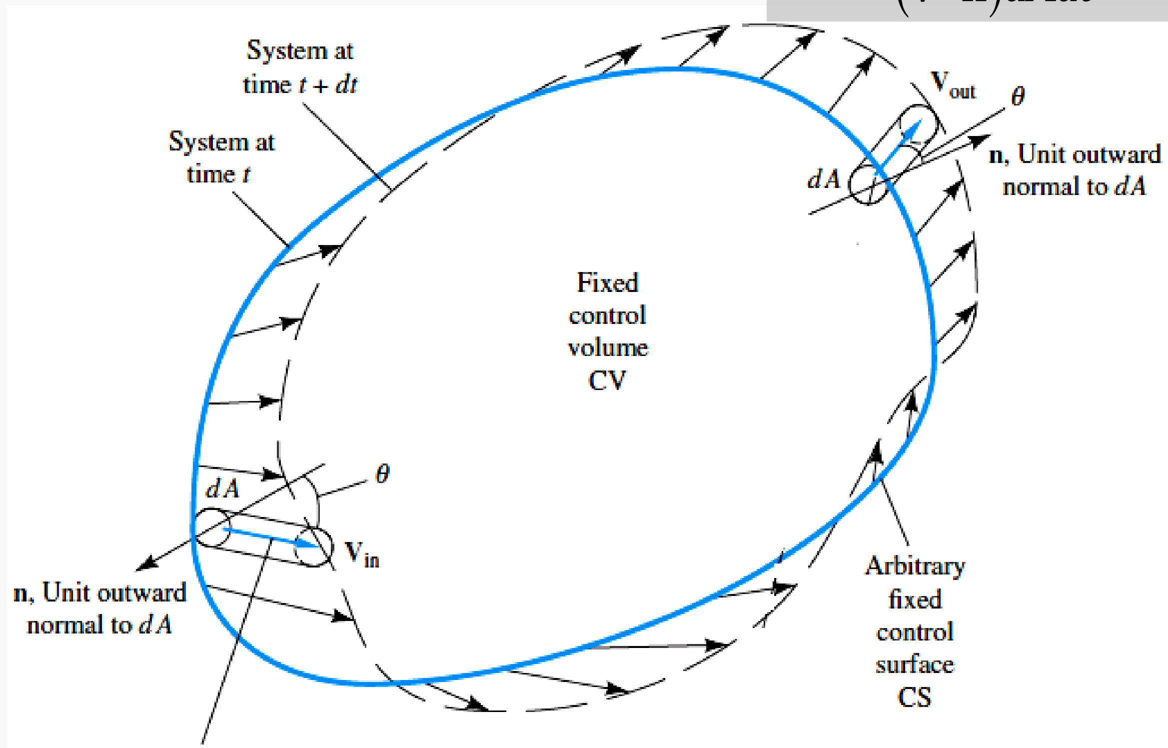
$$\beta = \frac{dB}{dm}$$

$$\rho = \text{density of fluid}$$

$$d\forall = \text{elemetal volume inside the CV}$$

Flow out +ve

$$d\forall_{out} = V_{out} dA_{out} \cos\theta_{out} dt = (\vec{\mathbf{V}} \cdot \hat{\mathbf{n}}) dA dt$$



$$d\forall_{in} = V_{in} dA_{in} \cos\theta_{in} dt = -(\vec{\mathbf{V}} \cdot \hat{\mathbf{n}}) dA dt$$

Flow in -ve

Situation at time *t+dt*

Reynolds Transport Theorem (RTT)

There are **three sources of change** in B relating to the control volume (CV):

1. A change within the control volume (CV):

$$\frac{dB_{CV}}{dt} = \frac{d}{dt}\left(\int_{CV} \beta\,\rho\, d\forall\right)$$

2. Outflow of β from the control volume through the control surface (CS):

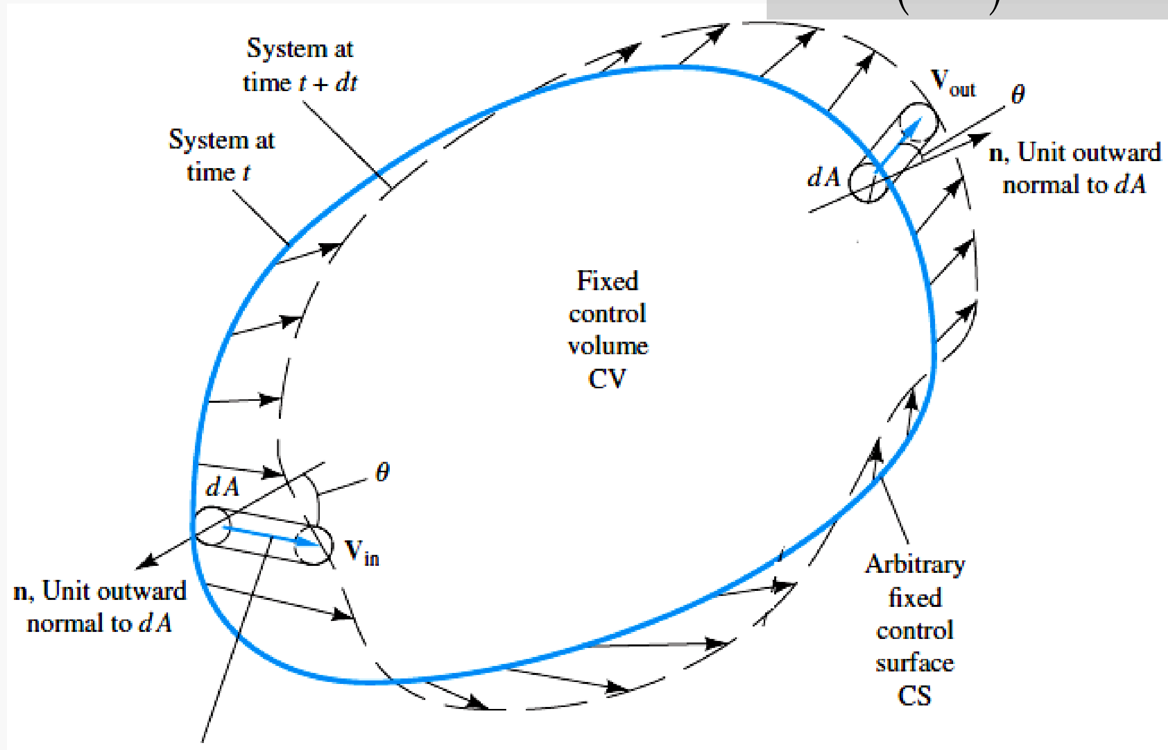
$$\int_{CS} \beta\,\rho V \cos\theta\, dA_{out}$$

3. Inflow of β to the control volume through the control surface (CS):

$$\int_{CS} \beta\,\rho V \cos\theta\, dA_{in}$$

Flow out +ve

$$d\forall_{out} = V_{out} dA_{out} \cos\theta_{out} dt = \left(\vec{\mathbf{V}}\cdot\hat{\mathbf{n}}\right) dA\, dt$$



$$d\forall_{in} = V_{in} dA_{in} \cos\theta_{in} dt = -\left(\vec{\mathbf{V}}\cdot\hat{\mathbf{n}}\right) dA\, dt$$

Flow in -ve

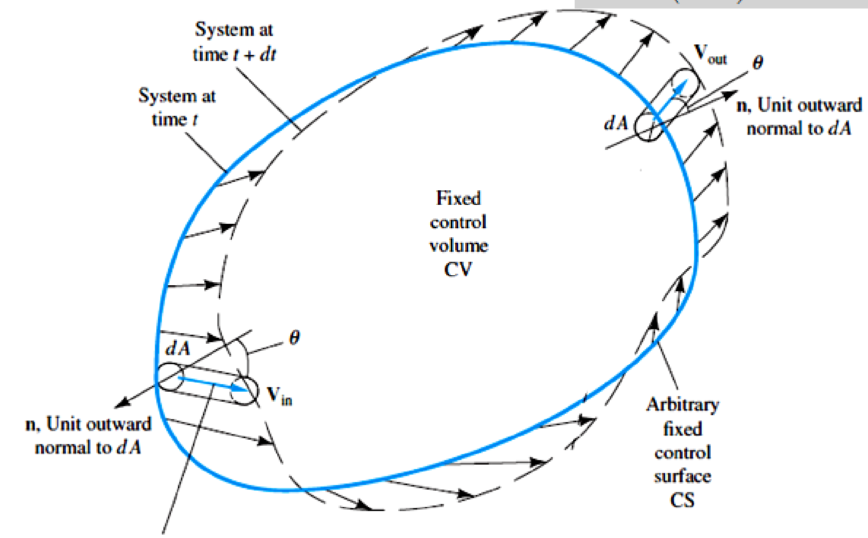
Situation at time *t+dt*

Reynolds Transport Theorem (RTT)

From the Fig., it is seen that the system has moved a bit, gaining the outflow sliver and losing the inflow sliver. In the limit as *dt* → 0, the instantaneous change of *B* in the ***system*** is the sum of the change within, plus the outflow, minus the inflow:

Flow out +ve

$$d\forall_{out} = V_{out} dA_{out} \cos\theta_{out} dt = \left(\vec{\mathbf{V}} \cdot \hat{\mathbf{n}}\right) dA dt$$



$$d\forall_{in} = V_{in} dA_{in} \cos\theta_{in} dt = -\left(\vec{\mathbf{V}} \cdot \hat{\mathbf{n}}\right) dA dt$$

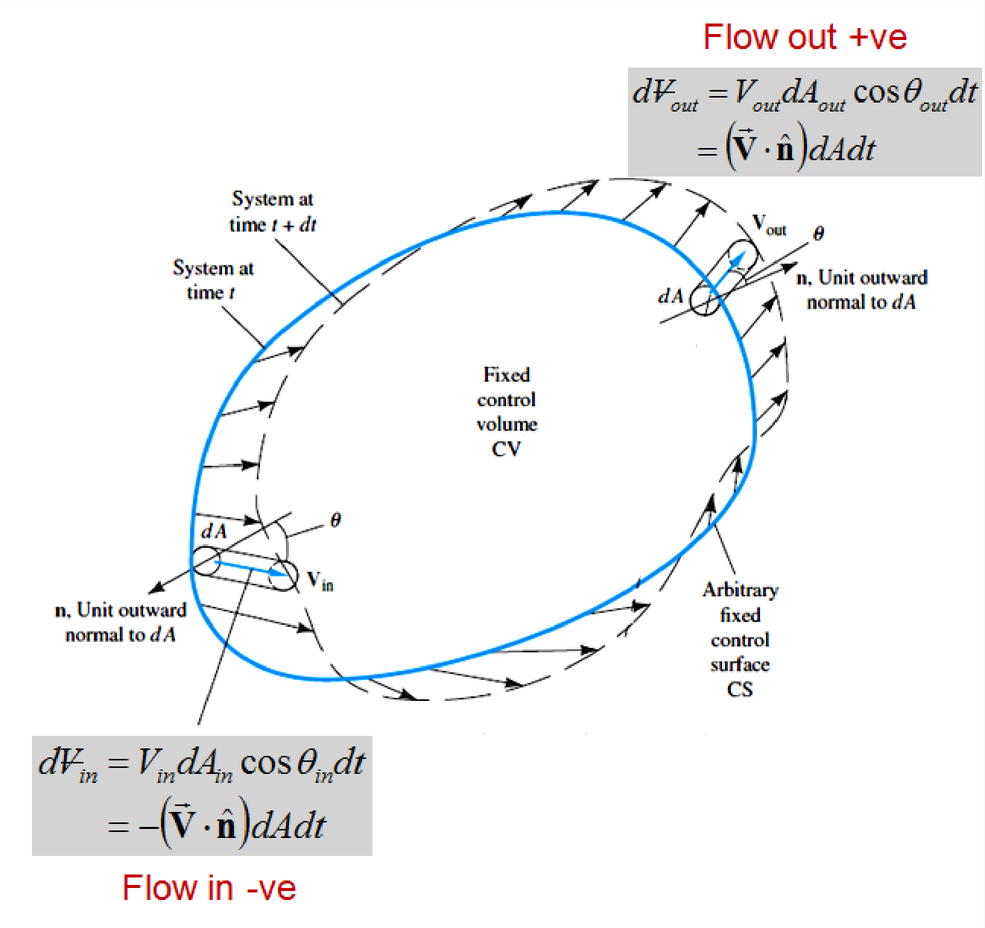
Flow in -ve

$$\boxed{\frac{d}{dt}\left(B_{\text{syst}}\right) = \frac{d}{dt}\left(\int_{CV} \beta\, \rho d\forall\right) + \underbrace{\int_{CS} \beta\, \rho V \cos\theta\, dA_{out} - \int_{CS} \beta\, \rho V \cos\theta\, dA_{in}}_{\text{Flux terms}}}$$

Reynolds Transport Theorem (RTT)

Now, the flux terms can be combined in a single integral term involving $\vec{\mathbf{V}}\cdot\hat{\mathbf{n}}$ that accounts for both positive outflow and negative inflow:

$$\text{Flux terms} = \int_{CS} \beta\,\rho V\cos\theta\, dA_{out} - \int_{CS} \beta\,\rho V\cos\theta\, dA_{in}$$
$$\equiv \int_{CS} \beta\,\rho\left(\vec{\mathbf{V}}\cdot\hat{\mathbf{n}}\right)dA$$



Thus the relation comes as:

$$\frac{d}{dt}\left(B_{syst}\right) = \frac{d}{dt}\left(\int_{CV} \beta\,\rho\, d\forall\right) + \int_{CS} \beta\,\rho\left(\vec{\mathbf{V}}\cdot\hat{\mathbf{n}}\right)dA$$

This is the general form of the Reynolds transport theorem (RTT) for a fixed, nondeforming control volume.

This relation permits to change from a system approach to control volume (CV) approach.

Reynolds Transport Theorem (RTT)

$$\frac{d}{dt}\left(B_{\text{syst}}\right) = \frac{d}{dt}\left(\int_{\text{CV}} \beta\, \rho\, d\forall\right) + \int_{\text{CS}} \beta\, \rho \left(\vec{\mathbf{V}} \cdot \hat{\mathbf{n}}\right) dA$$

Time rate of change of an arbitrary parameter, *B* of the system.

The parameter *B* may be mass, momentum, energy, or angular momentum etc.

Time rate of change of *B* within the control volume (CV) as the fluid flows through it.

β is the amount of *B* per unit mass.

Net flowrate of the parameter *B* across the entire control surfaces (CS).

The net flowrate across the entire control surfaces may be negative, zero, or positive depending on the particular situation involved.

Unsteady effects associated with the fact that the time rate of change of system property.

=

Unsteady effects associated with the fact that the values of the parameter within the control volume may change with time.

+

Convective effects associated with the flow of the system across the fixed control surfaces.

Recap

Reynolds transport theorem (RTT) for a fixed, nondeforming control volume (CV)

$$\boxed{\frac{d}{dt}\left(B_{\text{syst}}\right) = \frac{d}{dt}\left(\int_{\text{CV}} \beta\, \rho\, d\forall\right) + \int_{\text{CS}} \beta\, \rho \left(\vec{\mathbf{V}} \cdot \hat{\mathbf{n}}\right) dA}$$

This relation permits to change from a system approach to control volume (CV) approach.

where

B_{syst} = any property of fluid (mass, momentum, enthalpy, etc.)

β = intensive property of fluid (per unit mass basis)

ρ = density of fluid

$d\forall$ = elemental volume

$\left(\vec{\mathbf{V}} \cdot \hat{\mathbf{n}}\right) dA$ = elemental volume flux

$\int_{\text{CV}}$ = volume integral over the control volume (CV)

$\int_{\text{CS}}$ = surface integral over the control surface (CS)

Similar expression adopted by other books:

$$\boxed{\frac{D}{Dt}\left(B_{\text{syst}}\right) = \frac{\partial}{\partial t}\left(\int_{\text{CV}} \beta\, \rho\, d\forall\right) + \int_{\text{CS}} \beta\, \rho \left(\vec{\mathbf{V}} \cdot \hat{\mathbf{n}}\right) dA}$$

Conservation of Mass

A **system** is defined as a fixed quantity of mass, denoted by *m.* Thus, the mass of the system is conserved and does not change [except nuclear reaction] . so the **conservation of mass principle** for a system is simply stated as

$$m_{\text{syst}} = \text{const.}$$

$$\therefore \frac{dm_{\text{syst}}}{dt} = 0 \qquad (i)$$

Reynolds transport theorem (RTT) with *B* = mass and so, *β* = 1; accordingly

$$\beta = \frac{\text{mass}}{\text{mass}} = 1$$

$$\frac{d}{dt}\left(B_{\text{syst}}\right) = \frac{d}{dt}\left(\int_{CV} \beta\, \rho\, d\forall\right) + \int_{CS} \beta\, \rho\left(\vec{\mathbf{V}}\cdot\hat{\mathbf{n}}\right)dA$$

$$\Rightarrow \frac{d}{dt}\left(m_{\text{syst}}\right) = \frac{d}{dt}\left(\int_{CV} \rho\, d\forall\right) + \int_{CS} \rho\left(\vec{\mathbf{V}}\cdot\hat{\mathbf{n}}\right)dA$$

$$\boxed{\Rightarrow \frac{d}{dt}\left(\int_{CV} \rho\, d\forall\right) + \int_{CS} \rho\left(\vec{\mathbf{V}}\cdot\hat{\mathbf{n}}\right)dA = 0}$$

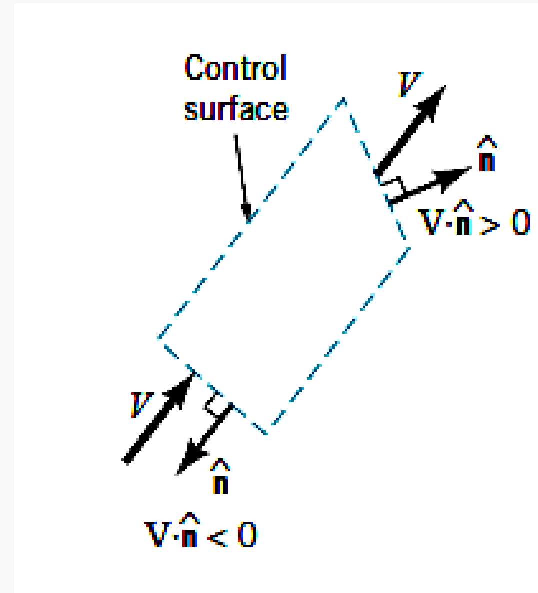
Control volume expression for conservation of mass, commonly known as **continuity equation**.

Conservation of Mass

For steady flow i.e. $\frac{d}{dt}(\ )=0$

$$\overbrace{\cancel{\frac{d}{dt}\int_{CV}\rho\, d\forall}}^{=\,0} + \int_{CS}\rho\left(\vec{\mathbf{V}}\cdot\hat{\mathbf{n}}\right)dA = 0$$

$$\boxed{\Rightarrow\ \int_{CS}\rho\left(\vec{\mathbf{V}}\cdot\hat{\mathbf{n}}\right)dA = 0 \qquad (ii)}$$



The integrand in the mass flow rate integral represents the product of the component of velocity, **V** perpendicular to the small portion of the control surface and the differential area, dA.

As shown in figure (dot product)

$\left(\vec{\mathbf{V}}\cdot\hat{\mathbf{n}}\right) = +\,\text{ve}$; **+ve for flow out** from the control volume

$\left(\vec{\mathbf{V}}\cdot\hat{\mathbf{n}}\right) = -\,\text{ve}$; **-ve for flow in** to the control volume

Equation (ii) states that in steady flow, the mass flows entering and leaving the control volume (CV) must balance exactly.

Conservation of Mass

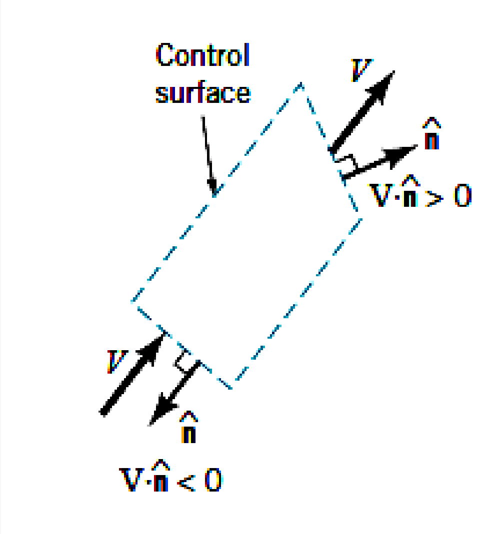
When all of the differential quantities are summed over the entire control surfaces;

$$\int_{CS} \rho\left(\vec{\mathbf{V}} \cdot \hat{\mathbf{n}}\right) dA = 0 \quad \equiv \sum(\rho A V)_{\text{out}} - \sum(\rho A V)_{\text{in}}$$

$$= \sum \dot{m}_{\text{out}} - \sum \dot{m}_{\text{in}} \quad = 0$$

$$\boxed{\Rightarrow \sum \dot{m}_{\text{in}} = \sum \dot{m}_{\text{out}}}$$

Mass continuity equation



For incompressible flows, (ρ =constant through the flow system)

$$\Rightarrow \sum(AV)_{\text{in}} = \sum(AV)_{\text{out}}$$

$$\boxed{\Rightarrow \sum Q_{\text{in}} = \sum Q_{\text{out}}}$$

volume continuity equation